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INVITATION

Following the acceptance of your paper “A New Algebraically Simple Five-MinimumTerm Approach to an Existing FDNR-Based Chaotic Circuit and Its New Homoclinic Orbit” at the 2016 International Symposium on Fundamentals of Electrical Engineering (ISFEE 2016), organized in our faculty on June 30th - July 2nd, 2016, you are welcome in Bucharest, Romania, to participate to this event.

DEAN,  
Dr. Ing. Dragoș NICULAE  

June 10, 2016
Abstract—A new algebraically simple five-minimum-term approach to an existing FDNR-based chaotic circuit is presented. An existing piecewise-linear model of a diode is replaced with a new better model using a conventional diode equation. Such a new model results in algebraically simple five minimum terms in three coupled ordinary differential equations (ODEs). Not only are the ODEs reduced from six to five minimum algebraic terms, but also from two nonlinear terms to a single nonlinear term. Better versions of chaotic attractors, a new bifurcation diagram and a new largest Lyapunov exponent are depicted. In particular, a new homoclinic orbit of the circuit is illustrated.

Index Terms—FDNR-based Chaotic Circuit, Homoclinic Orbit.

I. INTRODUCTION

Since 1963, the discovery of the Lorenz attractor [1] has attracted great attention to the studies of chaos in science and technology as well as in chaotic circuits and systems. Various applications of chaotic circuits include e.g. chaos-based secure communications [2]. Several techniques of chaotic circuits have been proposed e.g. [3]−[6], including a technique based on a frequency dependent negative resistor (FDNR). Early examples of FDNR-based chaotic circuits were reported in e.g. [7], [8], where a FDNR was employed as an active linear building block to demonstrate a source of circuit energy, whereas a diode was a passive nonlinear element. The diode was however not modeled by a conventional diode equation, but was approximately modeled by a piecewise-linear equation. This resulted in either ten algebraic terms [7] in four coupled ordinary differential equations (ODEs), or six algebraic terms [8] in three coupled ODEs of the circuit. In addition, such ODEs had two nonlinear terms [7], [8].

Although FDNR-based chaotic circuits have been suggested for more than a decade, publications on such chaotic circuits have been relatively rare. One of the main reason for such rare publications is probably that the existing FDNR-based chaotic circuits do not offer other obvious advantages over other existing approaches. Recently, a five-term chaotic attractor, e.g. [9], has been reported and the minimum number of algebraic terms in three coupled ODEs has been identified to be five terms. Such minimum terms have offered an advantage of simplicity. As the ODEs of the existing FDNR-based chaotic circuit [8] have six algebraic terms using the piecewise-linear model for the diode, it is natural to wonder whether [8] may have five minimum terms using a new model by a conventional diode equation.

In this paper, a new algebraically simple five-minimum-term approach to the existing FDNR-based chaotic circuit [8] is proposed. The diode is newly modeled by a conventional diode equation replacing the existing piecewise-linear model. This results in new five minimum algebraic terms in three coupled ODEs with only single nonlinearity. New dynamical results are presented including a new homoclinic orbit.

II. AN EXISTING FDNR-BASED CHAOTIC CIRCUIT USING A PIECEWISE-LINEAR MODEL

An existing FDNR-based chaotic circuit using a piecewise-linear model [8] is shown in Fig. 1(a), whereas Fig. 1(b)

![Fig. 1. (a) An existing FDNR-based chaotic circuit [8], (b) An equivalent circuit where the FDNR is denoted $Z_1$ on the right hand side of node $A$.](image-url)
shows the equivalent circuit. The circuit consists of an inductor $L$, a capacitor $C$, a diode $D$ and a FDNR denoted as $Z_1$ on the right hand side of node $A$, as shown in Fig. 1(b). This section briefly summaries such an existing FDNR-based chaotic circuit.

### A Frequency Dependent Negative Resistor (FDNR)

As shown in Fig. 1(a), the FDNR $Z_1$ on the right hand side of node $A$ consists of two op-amps $U_1$ and $U_2$, two capacitors $C_1$ and $C_2$, and three resistors $R_1$, $R_2$ and $R$. Let $R_1 = R_2 = R = R_F$, $C_1 = C_2 = C_F$, $K = R/R_F = 1$, impedance $Z_{C_1} = 1/(j\omega C_1)$, $Z_{C_2} = 1/(j\omega C_2)$, and the complex frequency $s = j\omega$. The FDNR $Z_1$ can be expressed as the form:

$$Z_i = R \left( \frac{Z_{C_1}Z_{C_2}}{R_1R_2} \right) = R \left( \frac{-1}{\alpha^2 C_2^2 R_F^2} \right) = \frac{1}{D_i s}$$  \hspace{1cm} (1)

where the constant $D_i = R_F C_2^2 / K$.

### B. An Existing Piecewise-Linear Model of the Circuit

Three coupled ODEs of the circuit shown in Fig. 1(a) are described by:

$$\begin{align*}
D_i \dot{v}_c &= -i_F \\
C \dot{v}_c &= i_F - i_k - I_N \\
L \dot{i}_k &= v_c
\end{align*}$$  \hspace{1cm} (2)

where the current $I_N$ through the diode $D$ is approximately described by a piecewise-linear equation of the form

$$I_N = \frac{1}{R_D} \begin{cases}
  V_f, & v_c \geq V_f \\
  0, & v_c < V_f
\end{cases}$$  \hspace{1cm} (3)

and $R_D$ is the diode forward conduction resistance ($\approx 50 \Omega$), and $V_f$ is the voltage drop ($\approx 0.6 \text{ V}$). Let normalized variables $(X, Y, Z)$, normalized constants ($\alpha, \beta$), normalized time $\tau$, and normalized derivative $(\dot{X}, \dot{Y}, \dot{Z})$ be defined as

$$\begin{bmatrix}
\dot{X} \\
\dot{Y} \\
\dot{Z}
\end{bmatrix} = \begin{bmatrix}
\alpha \frac{dX}{d\tau} \\
\beta \frac{dY}{d\tau} \\
-\alpha K \frac{dZ}{d\tau}
\end{bmatrix} + \begin{bmatrix}
\frac{v_c}{V_f} - R_F / R_D \\
\frac{R_i}{R_F} V_c \\
\frac{1}{C F}
\end{bmatrix}$$  \hspace{1cm} (4)

In this case, $Z = \dot{X}$. Based on (4), a normalized dimensionless version of (2) using (3) is a piecewise-linear dynamical model of the form

$$\begin{bmatrix}
\dot{X} \\
\dot{Y} \\
\dot{Z}
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 1 \\
\beta & 0 & 0 \\
-aK & -K & -K
\end{bmatrix} \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}$$  \hspace{1cm} (5)

where $a'\alpha$ is a nonlinear term of the form

$$a = \begin{cases}
\alpha, & X \geq 1 \\
0, & X < 1
\end{cases}$$  \hspace{1cm} (6)

It can be noticed that the existing approach on the piecewise-linear model in (3) and (6) results in six algebraic terms in three coupled ODEs as shown in (5). In addition, there are two nonlinear terms of $a'$ in (5). Consequently, (5) is not considered to be algebraically simple.

### III. A NEW ALGEBRAICALLY SIMPLE FIVE-MINIMUM-TERM APPROACH USING A DIODE EQUATION

The current $I_N$ of the diode $D$ was approximately modeled by a piecewise-linear equation in (3). Alternatively, $I_N$ may be better modeled by a conventional diode equation of the form

$$I_S = I_s \left[ \exp \left( \frac{v_c}{n V_f} \right) - 1 \right] \equiv I_s \exp \left( \frac{v_c}{n V_f} \right)$$  \hspace{1cm} (7)

where $V_f$ is the thermal voltage, $n$ is a scaling factor and $I_s$ is the reverse saturation current. Fig. 2 shows a comparison of $I_N$ between the existing piecewise-linear model in (3), as shown by a dotted line, and the new diode-equation model in (7), as shown by a solid line.

![Fig. 2. A comparison of $I_N$ between the new (solid line) and the existing (dotted line) models of a diode described in (7) and (3), respectively.](image)

Let a new normalized constant $A = R_F I_s / (n V_f)$. A new normalized dimensionless version of (2) using (7) is expressed as a new algebraically simple five-minimum-term approach of the form

$$\begin{bmatrix}
\dot{X} \\
\dot{Y} \\
\dot{Z}
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 1 \\
\beta & 0 & 0 \\
-aK & -K & -K
\end{bmatrix} \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
-A e^{\tau}
\end{bmatrix}$$  \hspace{1cm} (8)

Unlike the existing six algebraic terms described in (5), the new approach in (8) is described by only five minimum algebraic terms. In addition, there is only one single nonlinear (exponential) term in (8), whereas the existing piecewise-linear approach in (5) has two nonlinear terms of $a'\alpha$. Consequently, the new approach in (8) is considered to be algebraically simple.

### IV. NUMERICAL RESULTS

Trajectories of the new algebraically simple five-minimum-term approach in (8) are numerically simulated using the Runge-Kutta integrator with a fixed step side of 0.01. Parameters $C_1 = C_2 = C_F = 10 \text{ nF}$, $C = 10 \text{ nF}$, $L = 10 \text{ mH}$, $R = \ldots$
\[ R_1 = R_2 = R_F = 2 \, k\Omega, \quad I_S = 6.2229 \, nA, \quad n = 1.9224, \quad V_T = 25.85 \, mV, \quad \text{and} \quad K = 1. \] Initial conditions of \((X, Y, Z) = (1, 0.5, 0).\)

### A. New Better Versions of Chaotic Attractors

For purposes of direct comparisons, chaotic attractors using the existing piecewise-linear approach in (5) and (6) are numerically depicted in Figs. 3(a)–3(c), whereas the corresponding chaotic attractors using the new algebraically simple five-minimum-term approach in (8) are numerically depicted in Figs. 3(d)–3(f) on \((X, −Y), (X, Z)\) and \((Y, Z)\) planes, respectively. Note that only Fig. 3(a) was previously reported in [8] (see Fig. 2 of [8]), whereas Figs. 3(b)–3(f) are new simulation results presented in this paper.

It can be observed from Fig. 3 that although chaotic attractors of both approaches are approximately resembled, they are however not exactly the same in some detail. The chaotic attractors in Figs. 3(d)–3(f) of the new approach show new better versions closely similar to the chaotic attractors obtained from PSpice simulation, whereas the corresponding chaotic attractors in Figs. 3(a)–3(c) of the existing approach show relatively different versions from those obtained from PSpice simulation. For example, Fig. 3(d) is better than Fig. 3(a) because Fig. 3(d) is much better similar to the chaotic attractor obtained from the PSpice simulation previously reported in [8] (see Fig. 4(b) of [8]). In addition, the reason for such better versions is because the proposed diode equation in (7) is a better model for the diode \(D\) compared to the existing piecewise-linear model in (3), as shown in Fig. 2.

### B. New Dynamical Results

Fig. 4 shows a new numerically bifurcation diagram of the maximum of \(X\) (\(X\)-max) versus the parameter \(K\) from 0 to 4. A period-doubling route to chaos is obvious in Fig. 4. Fig. 5 illustrates values of a largest Lyapunov exponent (LLE) versus the parameter \(K\) from 0 to 4. It can be seen that the chaotic regions shown in Fig. 4 are associated with the positive values of the LLE shown in Fig. 5.

### C. A New Homoclinic Orbit

The new algebraically simple five-minimum-term approach in (8) has an equilibrium point at \((X, Y, Z) = (0, 0, 0).\) The Jacobian matrix at the equilibrium is described as

\[
J = \begin{bmatrix}
0 & 0 & 1 \\
B & 0 & 0 \\
-Ae^X & -K & -K
\end{bmatrix}
\]  

(9)

Table I shows Examples 1 and 2 of eigenvalues \((\lambda_1, \lambda_2, \lambda_3)\) owing to two different values of \(C_F.\) Each example has a negative real eigenvalue \((\lambda_1)\) and a complex conjugate pair of eigenvalues \((\lambda_{2,3} = \alpha \pm j\beta)\) where \(\alpha\) is positive. The
equilibrium point is therefore a spiral saddle point with an index 2, i.e. the unstable (out-set) manifold has two spatial dimension. As \( |\lambda_1| > \alpha \), the Shil’nikov condition [5] for a proof of chaos is therefore satisfied. Example 1 represents a general case where the equilibrium point does not locate on the attractor, as shown in Figs. 3(d)–3(f). On the other hand, Example 2 represents a specific case where the equivalent point intersects the attractor, as shown in Fig. 6 by a homoclinic orbit [5]. Clearly, such an orbit connects the equivalent point to itself on an \((X, Y, Z)\) plane, i.e. the unstable (out-set) manifold intersects the stable (in-set) manifold for a homoclinic connection, and therefore chaos does exist [5].

**V. CONCLUSION**

The new algebraically simple five-minimum-term approach to the existing FDNR-based chaotic circuit has been suggested. The existing approximated model of the diode based on the piecewise-linear equation has been replaced with a new better model based on the traditional diode equation. In the three coupled ODEs of the circuit, the existing six algebraic terms have been reduced to the new five minimum algebraic terms, whereas the existing two nonlinear terms have been reduced to only one single nonlinear term. New better versions of chaotic attractors have been compared with those of the existing approach. The bifurcation diagram, the largest Lyapunov exponent, and the homoclinic orbit have been illustrated.

**REFERENCES**


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**TABLE I EXAMPLES 1 AND 2 OF EIGENVALUES**

<table>
<thead>
<tr>
<th>Example 1</th>
<th>Example 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C = 10 \text{ nF} )</td>
<td>( C = 10 \text{ nF} )</td>
</tr>
<tr>
<td>( C_\pi = 10 \text{ nF} )</td>
<td>( C_\pi = 8.8 \text{ nF} )</td>
</tr>
<tr>
<td>( \lambda_1 = -1.9999 )</td>
<td>( \lambda_1 = -2.0087 )</td>
</tr>
<tr>
<td>( \lambda_{2,3} = 0.5000 \pm 1.3229 )</td>
<td>( \lambda_{2,3} = 0.4362 \pm 1.2499 )</td>
</tr>
</tbody>
</table>

![Fig. 6. A numerical plot of a homoclinic orbit.](image-url)